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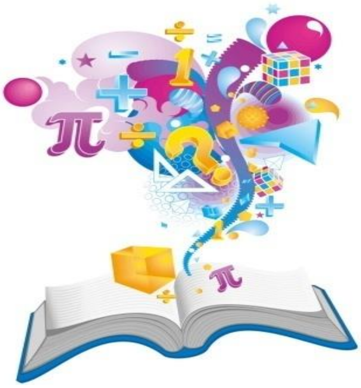
❖ Solutions to the above problems are invited, at the earliest. The names of the readers who turn out first in providing answers to the problems will be published and the solutions will be published in the forthcoming issue.



FROM THE EDITORIAL DESK

The PG Department of Mathematics has been established in the year 2003. It offers .Sc., Mathematics with Computer Application, B.Sc., Mathematics and M.Phil., Research :ogramme.

The department has to its credit, three National seminars, and an Intercollegiate meet ganisized on 11th & 12th August 2005, 30th & 31th August 2007, 9th January 2014 and 13th eptember 2011 respectively. It has celebrated National Mathematical year 2012 on 24th August)12. On memorial of Ramanujan's birthday Math Expo has organized by the department since)13.

The department is enriched with faculty members having wide knowledge in their ecializations like Differential Equations, Fuzzy Set Theory, Graph Theory, Operations Research c. The department has received two minor research projects funded by UGC. The department has :oduced 41 M.Phil., Research scholars from 2009 onwards.

The department adds one more feather by publishing a Subject Based Yearly News Letter corporating History of Mathematician, Crossword Puzzles, Cross out Crossword Puzzles, [athematics in Architecture, Solutions to the Problems of Previous issue, Department Activities id Placement Details of the Students of Mathematics.

We welcome the suggestions and criticism for improvement in the content and :esentation of materials of "MATH – MAZE".

EDITORIAL DESK

HISTORY OF REAL ANALYSIS

Mathematical analysis formally developed in the 17th century during the Scientific Revolution , but many of its ideas can be traced back to earlier mathematicians. Early results in analysis were implicitly present in the early days of ancient Greek mathematics. Later, Greek mathematicians such as Eudoxus and Archimedes made more explicit, but informal, use of the concepts of limits and convergence when they used the method of exhaustion to compute the area and volume of regions and solids.

The Indian mathematician Bhāskara II gave examples of the derivative and used what is now known as Rolle's theorem in the 12th century. Archimedes used the method of exhaustion to compute the area inside a circle by finding the area of regular polygons with more and more sides. This was an early but informal example of a limit, one of the most basic concepts in mathematical analysis.

In the 14th century, Madhava of Sangamagrama developed infinite series expansions, like the power series and the Taylor series, of functions such as sine, cosine, tangent and arctangent. Alongside his development of the Taylor series of the trigonometric functions, he also estimated the magnitude of the error terms created by truncating these series and gave a rational approximation of an infinite series. His followers at the Kerala school of astronomy and mathematics further expanded his works, up to the 16th century.

The modern foundations of mathematical analysis were established in 17th century Europe. Descartes and Fermat independently developed analytic geometry, and a few decades later Newton and Leibniz independently developed infinitesimal calculus, which grew, with the stimulus of applied work that continued through the 18th century, into analysis topics such as the calculus of variations, ordinary and partial differential equations, Fourier analysis, and generating functions. During this period, calculus techniques were applied to approximate discrete problems by continuous ones.

In the 18th century, Euler introduced the notion of mathematical function. Real analysis began to emerge as an independent subject when Bernard Bolzano introduced the modern definition of continuity in 1816, but Bolzano's work did not become widely known until the 1870s.

In 1821, Cauchy began to put calculus on a firm logical foundation by rejecting the principle of the generality of algebra widely used in earlier work, particularly by Euler. Instead, Cauchy formulated calculus in terms of geometric ideas and infinitesimals. Thus, his definition of continuity required an infinitesimal change in x to correspond to an infinitesimal change in y. He also introduced the concept of the Cauchy sequence, and started the formal theory of complex analysis. Poisson, Liouville, Fourier and others studied partial differential equations and harmonic analysis. The contributions of these mathematicians and others, such as Weierstrass, developed the (ε,δ)-definition of limit approach, thus give the way for the modern field of mathematical analysis.

In the middle of the 19^{th} century Riemann introduced his theory of integration. The last third of the century saw the arithmetization of analysis by Weierstrass, who thought that geometric reasoning was inherently misleading, and introduced the "epsilon-delta" definition of limit. Then, mathematicians started worrying that they were assuming the existence of a continuum of real numbers without proof. Dedekind then constructed the real numbers by which had already been developed by Simon Stevin in terms of decimal expansions. Around that time, the attempts to refine the theorems of Riemann integration led to the study of the "size" of the set of discontinuities of real functions.

Also, "monsters" (nowhere continuous functions, continuous but nowhere differentiable functions, space-filling curves) began to be investigated. In this context, Jordan developed his theory of measure, Cantor developed what is now called naive set theory, and Baire proved the Baire category theorem. In the early 20^{th} century, calculus was formalized using an axiomatic set theory. Lebesgue solved the problem of measure, and Hilbert introduced Hilbert spaces to solve integral equations

REAL ANALYSIS - BASIC DEFINITIONS

UPPER BOUND:

Let S be a set of real numbers. If there is a real number b such that $x \leq b$ for every x in S, then b is called an upper bound for S and S is bounded above by b.

SUPREMUM:

Let S be a set of real numbers bounded above. A real number b is called a least upper bound of S if it has the following two properties:

a) b is an upper bound for S.
b) No number less than b is an upper bound for S.

INFIMUM:

Let S be a set of real numbers bounded above. A real number a is called a lower bound for S if it has the following two properties:

a) a is a lower bound for S.
b) No number less than a is a lower bound for S.

COUNTABLE SET:

A set S is countable if it is either finite or countably infinite.

UNCOUNTABLE SET:

A set which is not countable is called uncountable.

INTERIOR POINT:

Let S be a subset of R^n, and assume that $a \in S$. Then a is called an interior point of S if there is an open n-ball with center at a , all of whose points belong to S**.**

OPEN SET:

A set S in R^n is called open if all its points are interior points.

CLOSED SET:

A set S in R^n is called closed if its complement $R^n - S$ is open.

ADHERENT POINT:

Let S be a subset of R^n, and x a point in R^n, x not necessarily in S. Then x is said to be adherent to S if every n - ball $B(x)$ contains at least one point of S.

ACCUMULATION POINT:

If $S \subseteq R^n$ and $x \in R^n$, then x is called an accumulation point of S if every n- ball $B(x)$ contains at least one point of S distinct from $\boldsymbol{x}$.

ISOLATED POINT:

If $x \in S$ but x is not an accumulation point of S, then x is called an isolated point of S.

BOUNDED SET:

A set S in R^n is said to be bounded if it lies entirely within an n -ball $B(a\,;\,r)$ for some $r > 0$ and some a in R^n.

COMPACT SET:

A set S in R^n is said to be compact if, and only if, every open covering of S contains a finite subcover, that is, a finite subcollection which also covers S.

CAUCHY SEQUENCE:

A sequence $\{x_n\}$ in a metric space (S, d) is called a Cauchy sequence if it satisfies the following condition (called the Cauchy condition) for every ε > 0 there is an integer N such that $d(x_n, x_m) <$ ε whenever $n \geq$ N and m ≥ N .

COMPLETE METRIC SPACES:

A metric spaces (S, d) is called complete if every Cauchy sequence in S converges in S. A subset T of S is called complete if the metric subspace (T, d) is complete.

CONTINUOUS FUNCTIONS:

Let (S, d_s) and (T, d_t) be metric spaces and let $f: S \to T$ be a function from S to T. The function f is said to be continuous at a point p in S if for every $\varepsilon > 0$ there is a $\delta > 0$ such that

$$d_t(f(x), f(p)) < \varepsilon \quad \text{whenever} \quad d_s(x , p) < \delta.$$

If f is continuous at every point of a subset A of S, we say f is a continuous on A.

UNIFORM CONTINUITY:

Let $f: S \to T$ be a function from one metric space (S, d_s) to another (T, d_T). Then f is said to be uniformly continuous on a subset A of S if the following conditions holds:

For every $\varepsilon > 0$ there exists a $\delta > 0$ (depending only on ε) such that if $x \in A$ and $p \in A$ then

$$d_T(f(x), f(p)) < \varepsilon \quad \text{whenever} \quad d_S(x , p) < \delta.$$

CONNECTEDNESS:

A metric space S is called disconnected if $S = A \cup B$, where A and B are disjoint nonempty open sets in S. We call S connected if it is not disconnected.

KNOW YOUR MATHEMATICIAN

Bernard Bolzano

Bernard Bolzano (1781–1848) was a catholic priest, a professor of the doctrine of catholic religion at the philosophical faculty of the University of Prague, an outstanding mathematician and one of the greatest logicians or even the greatest logician who lived in the long stretch of time between Leibniz and Frege.

In Mathematics he is not only known for his famous paradoxes of the infinite, but also for certain results those have become and still are standard in textbooks of Mathematics such as the Bolzano-Weierstrass theorem. Bolzano also made important contributions to other fields of knowledge in and outside of philosophy.

Bolzano entered the philosophy faculty of the Charles University of Prague in 1796, studying philosophy, physics and mathematics. Bolzano was deeply interested in the philosophy of Mathematics, and took great care to prove many results which were thought "obvious", so not requiring proof, by other mathematicians of the day.

During the year 1799-1800 Bolzano undertook research in Mathematics with Frantisek Josef Gerstner and also contemplated his future. In the autumn of 1800, against his father's wishes, he began three years of theological study at the Charles University. While pursuing his theological studies he prepared a doctoral thesis on geometry. He received his doctorate in 1804 writing a thesis giving his view of mathematics, and what constitutes a correct mathematical proof.

Bolzano began his mathematical investigations with a work in geometry (Bolzano 1804) and continued with publications in the area of analysis. He essentially formulated the modern criterion of convergence in his purely analytic proof already four years prior to Cauchy's course analyse (1821). This work already contains the so-called Bolzano-Weierstrass theorem, which again occurs in Bolzano's posthumously edited theory of functions. Here we find also for the first time an example of a continuous and still non-differentiable function.

Bolzano gives examples of one to one correspondences between the elements of an infinite set and the elements of a proper subset. Most of Bolzano's works remained in manuscript and did not become noticed and therefore did not influence the development of the subject. Many of his works were not published until 1862 or later. Bolzano's theories of mathematical infinity anticipated Georg Cantor's theory of infinite sets. It is also remarkable that he gave an example of a function which is nowhere differentiable yet everywhere continuous.

Karl Weierstress

Weierstrass was born in Ostenfelde, part of Ennigerloh, Province of Westphalia. Weierstrass was the son of Wilhelm Weierstrass, a government official, and Theodora Vonderforst. His interest in Mathematics began while he was a gymnasium student at Theodorianum in Paderborn. He was sent to the University of Bonn upon graduation to prepare for a government position. Because his studies were to be in the fields of law, economics, and finance, he was immediately in conflict with his hopes to study mathematics. He resolved the conflict by paying little heed to his planned course of study, but continued private study in mathematics. The outcome was to leave the university without a degree. After that he studied mathematics at the University of Munster (which was even at this time very famous for Mathematics) and his father was able to obtain a place for him in a teacher training school in Munster. Later he was certified as a teacher in that city. During this period of study, Weierstrass attended the lectures of Christoph Gudermann and became interested in elliptic functions. In 1843 he taught in Deutsch-Krone in Westprussia and since 1848 he taught at the Lyceum Hosianum in Braunsberg. Besides mathematics he also taught physics, botanics and gymnastics.

After 1850 Weierstrass suffered from a long period of illness, but was able to publish papers that brought him fame and distinction. He took a chair at the Technical University of Berlin, then known as the Gewerbeinstitut. He was immobile for the last three years of his life, and died in Berlin from pneumonia.

Weierstrass was interested in the soundness of calculus, and at the time, there were somewhat ambiguous definitions regarding the foundations of calculus, and hence important theorems could not be proven with sufficient rigour. While Bolzano had developed a reasonably rigorous definition of a limit as early as 1817 (and possibly even earlier) his work remained unknown to most of the mathematical community until years later, and many had only vague definitions of limits and continuity of functions.

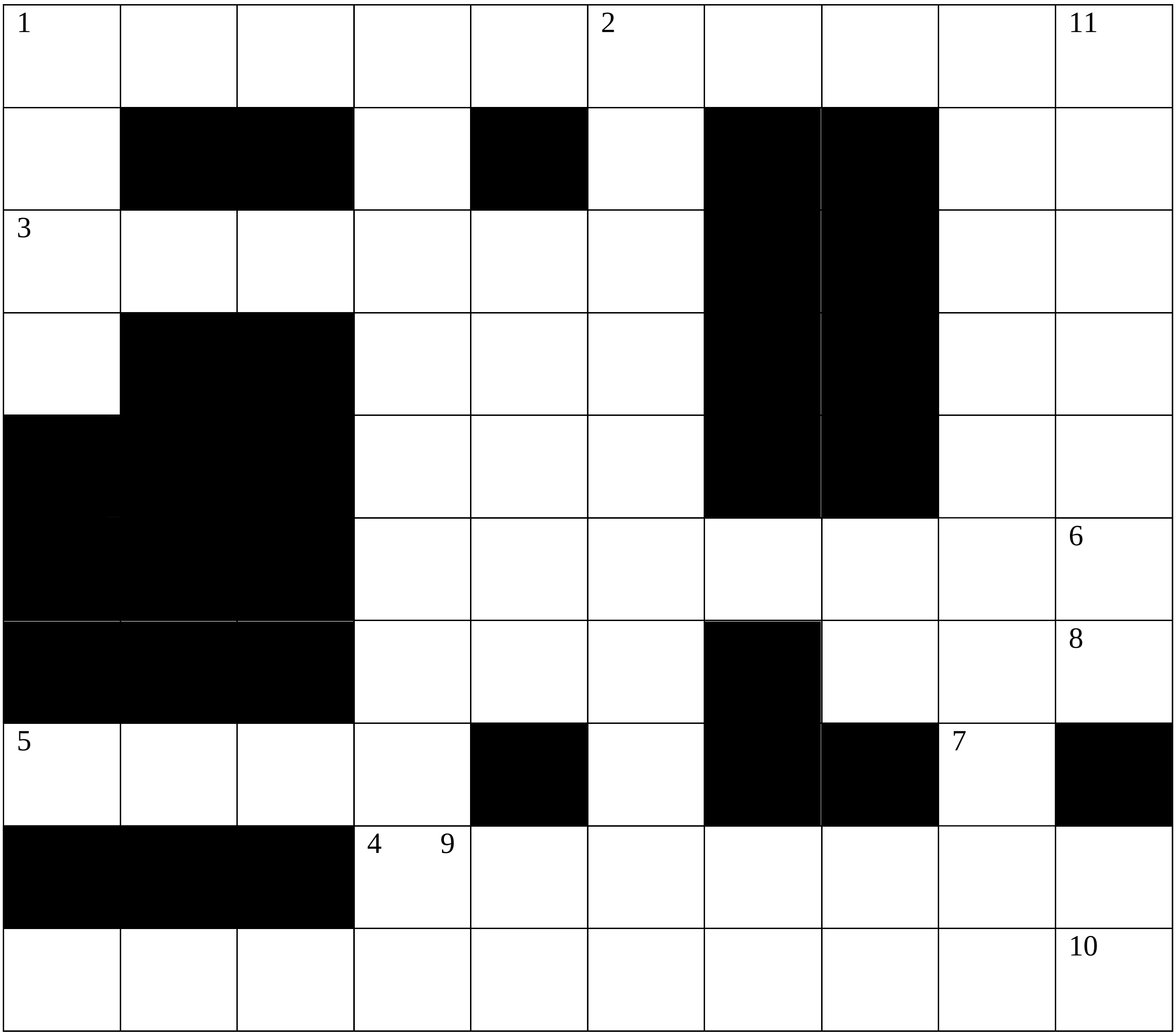
Delta-epsilon proofs are first found in the works of Cauchy in the 1820s. Cauchy did not clearly distinguish between continuity and uniform continuity on an interval. Notably, in his 1821 coursd'analyse, Cauchy argued that the limit of continuous functions was itself continuous, a statement interpreted as being incorrect by many scholars. The correct statement is rather that the uniform limit of continuous functions is continuous (also, the uniform limit of uniformly continuous functions is uniformly continuous).

The concept of uniform convergence, which was first observed by Weierstrass's advisor, Christoph Gudermann, in an1838 paper, where Gudermann noted the phenomenon but did not define it or elaborate on it. Weierstrass saw the importance of the concept, and both formalized it and applied it widely throughout the foundations of calculus.

Using the definition and the concept of uniform convergence, Weierstrass was able to write proofs of several unproved theorems such as the intermediate value theorem (for which Bolzano had already given a rigorous proof), the Bolzano-Weierstrass theorem, and Heine–Borel theorem.

Weierstrass also made significant advancements in the field of calculus of variations. Using the apparatus of analysis that he helped to develop, Weierstrass established a necessary condition for the existence of strong extrema of variational problems. He also helped devise the Weierstrass - Erdmann condition, which gives sufficient conditions for an extremal to have a corner along a given extrema, and allows one to find a minimizing curve for a given integral.

CROSSWORD PUZZLES



Top to Bottom:

1. The set of all interior point is ------------.

2. A set is closed iff it contains all its ------------ points.

11. If subsets satisfy $A \subseteq S \subseteq \overline{A}$ where $\overline{A}$ is the closure of A, then A is said to be ------------ in S.

Left to Right:

1. The set of all points x in R^n such that $\|x - a\| < r$ is called an ------------.

3. No elements in a set is called ------------ set.

5. The set of numbers of the form $\frac{1}{n}$, $n = 1,2,3, \ldots$, has ------------ as an accumulation point.

9. If a set is both closed and bounded then it is said to be ------------.

Bottom to Top:

4. A metric space (S, d) is called ------------ if every Cauchy sequence in S converges in S.

7. The set of ordered pairs is called ------------.

Right to Left:

6. Every finite subset of metric space is ------------.

8. ------------ is a collection of well defined objects.

10. In R^k, every Cauchy sequence is ------------.

CROSSOUT CROSSWORD PUZZLE

F	R	E	L	A	T	I	O	N	C	Q	J	G	L	M	D
T	C	H	P	R	N	A	U	B	O	P	E	N	N	B	H
H	O	C	I	E	L	M	G	F	U	J	C	S	V	V	U
L	M	P	A	N	W	E	D	Z	N	S	O	U	C	C	T
A	P	M	O	H	V	E	M	P	T	Y	N	P	O	A	N
N	L	T	B	L	Z	E	N	L	A	O	T	R	N	F	E
O	E	Q	D	P	O	L	R	J	B	E	I	E	S	J	G
I	T	E	U	S	J	G	W	S	L	H	N	M	T	P	R
T	E	U	B	B	I	Q	I	Q	E	C	O	U	A	I	E
A	Y	Q	C	O	M	P	A	C	T	Q	U	M	N	Y	V
R	Z	I	W	J	O	V	G	P	A	E	S	D	T	R	N
R	O	N	L	R	T	Y	Z	N	U	L	O	A	Z	W	O
I	E	U	C	L	I	D	E	A	N	M	E	T	R	I	C

1. Any set of ordered pairs is called a ______________.
2. If f is one-to-one on a subset S of $D(f)$, then the restriction of f to S has an ___________.
3. Every subset of a countable set is _____________.
4. $B - A = B$ if $B \cap A$ is _______________.

1. A set S is ___________ if and only if S = int S.
2. A set S in R^n is said to be ______________ iff every open covering of S contains a finite subcover, that is, a finite sub collection which also covers S.
3. $M = R^n$;$d\ (x, y) = \| x - y \|$. This is called ___________.
4. In Eucledian space R^k every Cauchy sequence is ____________.
5. Let $S = C$, the complex plane. Then ________ function, defined by *f(z)* = C for every z in C.
6. A property of a set which remains invariant under every topological mapping is called ______________ property.
7. A contraction of a complete metric space S has a ___________ fixed point p.
8. Let A be a subset of a metric space S. if A is ______________ then A is closed.
9. If f is differentiable at c, then f is ______________at c.
10. For sets which are bounded above but have no maximum element, there is a concept which takes the place of maximum element is called __________.
11. If $e^x = 1 + x + \frac{x^2}{2!} + \cdots\cdots \frac{x^n}{n!} + \cdots\cdots$ then the number e is ______________.

MATHEMATICS IN ARCHITECTURE

Tetrahedral - Shaped Church

The tetrahedron is a convex polyhedron with four triangular faces. Basically, it's a complex pyramid. You've seen the same geometric principle used in RPGs, because the dice is shaped the same. Famed architect Walter Netsch applied the concept to the United States Air Force Academy's Cadet Chapel in Colorado Springs, Colorado. It's a striking and classic example of modernist architecture, with its row of 17 spires and massive tetrahedron frame that stretches more than 150 feet into the sky. The early 1960's church cost a whopping $3.5 million to construct.

Experimental Math - Music Pavilion

Imagine walking up to the Philips Pavilion at the 1958 World's Fair and seeing this crazy construction of asymmetric hyperbolic paraboloids and steel tension cables.This amazing building appeared at the first Expo after World War II, so it was an important moment that allowed its creators to show off the technological progress the world had made since the devastating battle. Philips Electronics Company wanted to create a unique experience for visitors, so they collaborated with an international group of renown architects, artists, and composers to create the experimental space.

ArchDaily wrote about the groundbreaking, temporary building, calling it the “first electronic-spatial environment to combine architecture, film, light and music to a total experience made to functions in time and space. It was through these visually inspired concepts that elevated the Philips Pavilion into a complete experience where one could visualize their special movements through a space of sound, light, and time.” Poeme Electronique was one of the works prominently displayed at the time.

Solar Algorithm Wizardry

Barcelona’s Endesa Pavillion used mathematical algorithms to alter the cubic building’s geometry, based on solar inclination and the structure’s proposed orientation. Algorithms can be used to create the perfect building for *any* location with the right computer program. For Endesa, the movement of the sun was tracked on site before an architect from the Institute for Advance Architecture of Catalonia stepped in to complete the picture. The algorithm essentially did all the planning for him, calculating the building’s optimal form for that particular location.

"The science of mathematics presents the most brilliant example of how pure reason may

A Mathematically - Inclined Cucumber in the Sky

Standing 591-feet tall, with 41 floors is London's skyscraper known as The Gherkin (yes, like the cucumber). The modern tower was carefully constructed with the help of parametric modeling amongst other math formulas so the architects could predict how to minimize whirlwinds around its base. The design's tapered top and bulging center maximize ventilation. The building uses half the energy of other towers the same size. Any mathematician would be pleased to claim credit for the building, but architectural firm Foster and Partners might have something to say about that.

Magic Square Cathedral

The Sagrada Familia cathedral in Barcelona designed by Antoni Gaudí is a mathematician's dream. Hyperbolic paraboloid structures are featured throughout. Have you eaten Pringles? Then you definitely know what a Hyperbolic paraboloid structure is. Catenary arches (a geometric curve) abound. The cathedral also contains a Magic Square — an arrangement of numbers that equal the same amount in every column, row, and diagonal. The magic number in Sagrada Familia's case is 33, which alludes to multiple religious symbols. For example, Jesus performed 33 recorded miracles, and most Christians believe he was crucified at 33 years old in 33 A.D.

WEBSITES FOR TEACHING AND LEARNING

Website	Description
ptel.iitm.ac.in	NPTEL provides E-learning through online Web and Video courses in Engineering, Science and humanities streams. The mission of NPTEL is to enhance the quality of Engineering education in the country by providing free online courseware.
cademicearth.org	Academic Earth is a website launched March 24, 2009, by Richard Ludlow and co-founders Chris Bruner and Liam Pisano, which offers free online video lectures from universities such as UC Berkeley, UCLA, University of Michigan, Harvard, MIT, Princeton, Stanford, and Yale in the subjects of Astronomy, Biology, Chemistry, Computer Science, Economics, Engineering, English, Entrepreneurship, History, Law, Mathematics, Medicine, Philosophy, Physics, Political Science, Psychology, Religion, and Statistics.
areerstrokes.com	Motivates and inspires to guide you through a program of career building and personality development. Program designed for school students, college students, and young professionals for achieving success in life. An e-learning platform where it concentrates on Career, Personality, Goal Setting, Career Planning, Stress management etc.
dustrokes.com	An Online E-learning portal especially for kids and students who can learn their Maths and Science in a fun way by using Sports concept.
areervarsity.com	A Learning Portal for Career Aspiring Students from Bharathiar University.
ptitudecoach.com	A Portal for aptitude training and domain knowledge for Job-seekers
\cademy.com	It's a website to learn for free about math, art, computer programming, economics, physics, chemistry, biology, medicine, finance, history, and more. also provides a web-based exercise system that generates problems for students based on skill level and performance.

MATH GLOSSARY

Absolute value

The absolute value of an integer is its distance from zero on the number line.

Abundant number

An abundant number or excessive number is a number for which the sum of its proper divisors is greater than the number itself.

Acute angle

An angle that measures less than 90 degrees.

Acute triangle

An acute triangle has three angles that measure between 0 and 90 degrees.

Addend

Addends are numbers being added together.

Addition

Addition is the process of combining two or more numbers into one sum.

Addition Property of Equality

The property that states that if you add the same number to both sides of an equation, the sides remain equal (i.e., the equation continues to be true.)

Additive inverse

An additive inverse is the opposite of a given number.

Adjacent angles

Adjacent angles are angles that are side by side and have a common vertex and ray.

Algebra

Algebra is the branch of mathematics concerning the study of the rules of operations and relations, and the constructions and concepts arising from them, including terms, polynomials, equations and algebraic structures.

Algebraic equation

Algebraic equation is an equation that includes one or more variables.

Algebraic expression

An algebraic expression is a mathematical expression that consists of variables, numbers and operations. The value of this expression can change.

Algebraic numbers

An algebraic number is a number that is a root of a non-zero polynomial in one variable with rational coefficients.

Alphametic numbers

Alphametic numbers form cryptarithms where a set of numbers are assigned to letters that usually spell out some meaningful thought

Amicable numbers

Amicable numbers are pairs of numbers, each of which is the sum of the others aliquot divisors

Angle

An angle is a figure formed by two rays that have a common endpoint.

Angle measure

The size of an angle is measured in degrees.

Arc

An arc is a part of a circle named by its endpoints.

Area

Area is defined as the number of square units covers a closed figure.

Area of a circle

The area of circle is the number of square units inside that circle.

Area of a polygon

The area of a polygon is the number of square units inside that polygon.

Arithmetic

The branch of mathematics usually concerned with the four operations (addition, subtraction, multiplication and division) of positive numbers.

Arithmetic expression

An algebraic expression is a mathematical expression that consists of variables, numbers and operations. The value of this expression can change.

Arithmetic mean

The arithmetic mean (or simply the mean) of a list of numbers is the sum of all of the list divided by the number of items in the list.

Arithmetic operations

The four basic arithmetic operations are addition, subtraction, multiplication and division.

Arrangement numbers

Arrangement numbers, more commonly called permutation numbers, or simply permutations, are the number of things can be ordered or arranged.

Associative law of multiplication

The associative law of multiplication states that $(a\ b)\ c = a\ (b\ c)$.

Associative property

An operation is associative if you can group numbers in any way without changing the answer. It doesn't matter how you combine them, the answer will always be the same. Addition and multiplication are both associative.

Automorphic numbers

Automorphic numbers are numbers of "n" digits whose squares end in the number itself.

Average

The number obtained by dividing the sum of a set of numbers by the number of addends.

Axes

Axes are the horizontal number line (x-axis) and the vertical number line (*y*-axis) on the coordinate plane. Axes are also the lines at the side and bottom of a graph.

SOLUTIONS TO THE PROBLEMS OF THE PREVIOUS ISSUE

CROSSWORD PUZZLES – COMPLEX ANALYSIS

ANSWERS:

LEFT TO RIGHT

1. Meromorphic
2. Closed
3. Isogonal
4. Fixed
5. Plane
6. Rectifiable

RIGHT TO LEFT

8. Cross
9. Rational

TOP TO BOTTOM

4. Finite
8. Constant
18. Inverse
19. Harmonic

BOTTOM TO TOP

2. Compact
10. Open
11. Critical
12. Limit
13. Annular
14. Absolute
15. Entire
16. Real
17. Zero

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π - MATH ASSOCIATION

MATH - MAZE

Bernard Bolzano
(1781–1848)

Karl Weierstrass
(1815 - 1897)

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